

LATERAL LOAD TESTS ON DRILLED PIERS IN SAND

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ABSTRACT

Results of full-scale lateral load tests are presented for drilled and cast-in-place piers with diameters of 2 ft to 4 ft (0.61 m to 1.22 m) and lengths of 17 ft and 18 ft (5.18 m and 5.49 m). Maximum lateral loads of up to 200 kip (890 kN) were applied, and lateral deflections were measured. Using available procedures, the observed load-deflection behavior was compared to predicted behavior. A simple, empirical method for predicting lateral load-deflection response is proposed.

INTRODUCTION

Drilled piers have been used extensively for supporting axial and lateral loads for a variety of structures including buildings, bridges, highway structures, and transmission towers. Lateral loads govern the design of piers in many cases. Pier design for lateral load can be based on ultimate load analyses and a factor of safety, or on an allowable lateral deflection. A design based on an allowable lateral deflection provides a more rational approach because it can allow the design to incorporate the deflection tolerance of the structure. A number of methods are available to predict the lateral load behavior of piers (5,7,9,14,16), but only a few full-scale load test results are reported in the literature (1,4,8) for drilled piers in sand.

This paper presents the results of full-scale lateral load tests on seven drilled piers in medium dense to very dense sands and compares the observed and predicted behavior. The tests were conducted on piers with dia-

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meters of 2 ft, 3 ft, 3.5 ft, and 4 ft (1 ft= 0.305 m) and lengths of 17 ft and 18 ft (5.18 m and 5.49 m). Piers were constructed at three sites. Lateral loads up to 200 kip (890 kN) were applied, and measurements of groundline deflection versus load were made for all piers. Comparisons between the observed and predicted behavior using existing published procedures show that predicted deflections are generally two to five times greater than the observed deflections. A new procedure is proposed for predicting lateral deflections of drilled piers in sand.

PIER AND SOIL CONDITIONS

Seven cast-in-place drilled piers were constructed and tested for the Southern California Edison Company (SCE). The pier characteristics are shown in Table 1. Piers 1, 2, and 3 were constructed and tested in 1963 at a site (Site A) located about 42 miles (67.6 km) east of downtown Los Angeles(6). Piers 4-7 were constructed and tested in 1979 at two locations (Sites B and C) near Daggett, California. Each pier was constructed by drilling the pier holes in the dry with an auger, lowering the reinforcing cage in the open hole, and placing the concrete.

TABLE 1.-- Pier Characteristics

Pier number (1)	Nominal diameter in feet (2)	Embedded length in feet (3)	Test site (4)	Concrete modulus of elasticity in psi (5)	Reinforcement (6)
1	3.5 ^a	17.0	A	3 x 10 ⁶	not known
2	3.5 ^a	17.0	A	3 x 10 ⁶	not known
3	3.5 ^a	17.0	A	3 x 10 ⁶	not known
4	2.0	18.0	B	4.33 x 10 ⁶	14 #11 bars
5	3.0	18.0	B	4.33 x 10 ⁶	14 #11 bars
6	3.0	18.0	C	4.33 x 10 ⁶	14 #11 bars
7	4.0	18.0	C	4.33 x 10 ⁶	14 #11 bars

^aPiers were constructed with a 5-ft diameter bell in the bottom 2 ft.

Note: 1 ft= 0.305 m, 1 psi= 6.9 kN/m²

Soil conditions at Site A were investigated by drilling a soil boring and measuring penetration resistance. For Sites B and C, soil conditions were determined by drilling a soil boring at each site, performing standard penetration tests and static cone penetration tests, and performing laboratory strength and classification tests on

intact samples. Standard penetration blow counts and static cone penetration resistance are shown in Figs. 1 and 2, respectively.

The soils at Site A are predominantly sands ranging in size from fine to coarse, with scattered gravels and cobbles. The relative density increases with depth from loose near the surface to dense at depth. The soils at Sites B and C are also generally fine to coarse-grained sands and silty sands with variable amounts of gravels below 4 ft (1.22 m). The natural moisture content varies between 28 and 10% with an average value of 4%, and the dry density varies between 81 pcf and 113 pcf (1335 kg/m³ and 1809 kg/m³) with an average value of 99 pcf (1585 kg/m³). The effective angle of internal friction for the near-surface silty sands was found to be 36° from triaxial compression tests with confining pressures in the range of 1 to 10 ksf (47.9 to 479 kN/m²).

The relative densities of sands were estimated by using the standard penetration resistances (N-values) and the correlation proposed by Bazarra(3). The average values of soil parameters used in analyses are shown in Table 2. Where laboratory data are not available, values shown are based on correlations with N-values and cone penetration resistances.

LOAD TEST PROCEDURE

Piers 1 - 3 were constructed in a triangular configuration with a center-to-center distance of about 29 ft. The loading was applied by forcing the piers apart using a 200 kip (890 kN) hydraulic jack. The load was applied at a point about 15 in. (381 mm) below the ground surface. Independently supported dial gages, reading to 0.001 in. (0.0254 mm), were placed in direct contact with the piers to obtain groundline deflection. Variation of lateral deflection with depth was recorded using a slope indicator for several loads. Two tests were made, Test 1 between Piers 1 and 3 and Test 2 between Piers 2 and 3.

The loads were applied in increments of 5 to 25 kip (22.2 to 111.2 kN), and groundline displacements were recorded for each increment. Where creep occurred, the load was maintained constant, and final displacement was obtained when the rate of displacement was less than 0.0005 in./min (0.0127 mm/min).

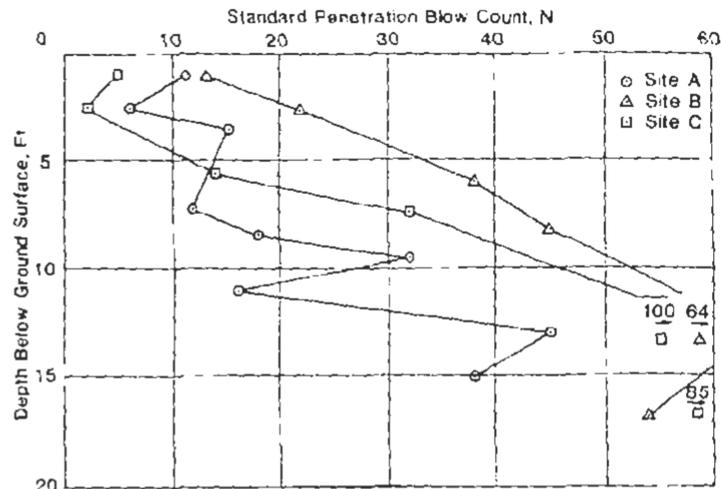


Figure 1. BLOW COUNT VERSUS DEPTHS, SITES A, B AND C. (1 ft = 0.305 m)

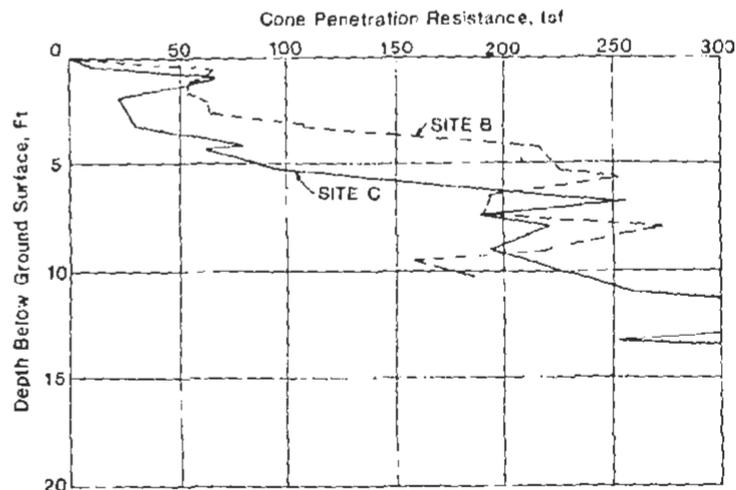


Figure 2. CONE PENETRATION RESISTANCE, SITES B AND C (1 ft = 0.305 m, 1 tsf = 95.8 kN/m²)

TABLE 2. -- Summary of Soil Data

Site number (1)	Soil type (2)	Depth in feet (3)	Total unit weight in pounds per cubic foot (4)	Friction angle ϕ , in degrees (5)	Relative density D_r as a percentage (6)	Pier test (7)
A	Sand (SP-SM)	0-8	105	38	55	1, 2, 3
	Sand (SP-SM)	8-15	110	40	67	
B	Silty sand (SM)	0-3	105	36	77	4, 5
	Silty sand (SM) w/gravelly layers	3-18	105	42	88	
C	Silty sand (SM)	0-6	105	36	38	6, 7
	Silty sand (SM) w/gravelly layers	6-18	105	42	92	

Note: 1 ft = 0.305 m, 1 pcf = 16.01 kg/m³

Piers 4-5 and 6-7 were constructed in pairs at Site B and Site C, respectively, with a center-to-center spacing of about 20 ft (6.1 m). General load-test arrangement is shown in Fig. 3. The load was applied by a 200 kip (890 kN) hydraulic jack bearing on the jacking plate built into the pier. The point of application of the horizontal load was assumed to be at the level of the loading rod which was at the ground surface. Horizontal displacements of the piers were measured by one dial gage in front of the pier parallel to the loading rod at a height of about 6 in. (152 mm) above the ground surface.

Dial gages with a 0.001 in. (0.0254 mm) resolution and 2 in. (51 mm) travel were used to measure the horizontal movement; these gages were suspended from a frame which was supported independently at a minimum distance of 10 ft (3.05 m) from the piers. Loads were applied in 10-kip (44.5-kN) increments up to 100 kip (445 kN) with unloading after each increment. Cyclic loads

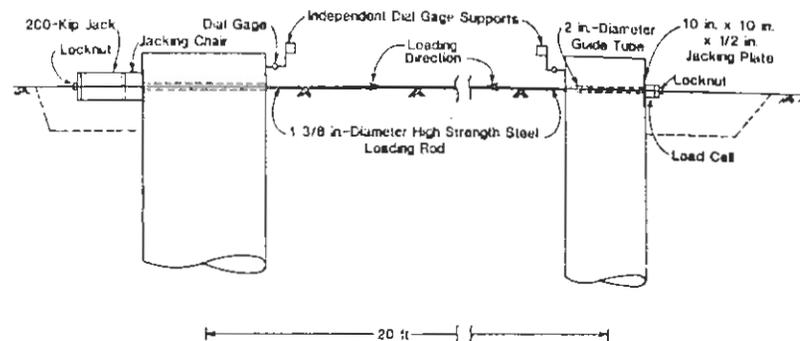


Figure 3. SCHEMATIC TEST SET UP - PIERS 4 TO 7
(1 ft = 0.305 m, 1 in. = 2.54 cm, 1 kip = 4.45 kN)

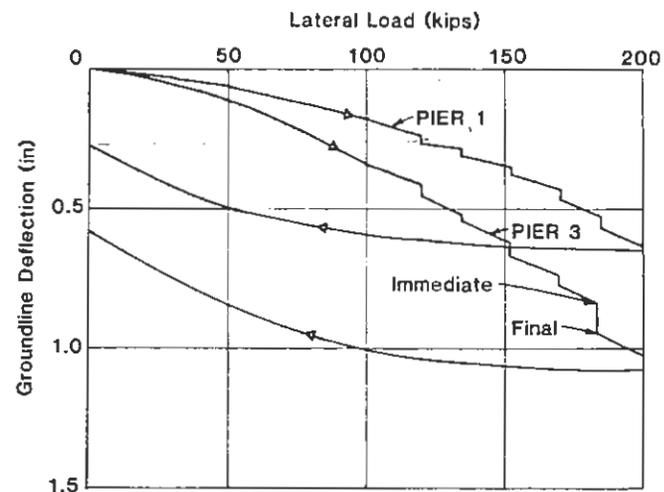


Figure 4. LOAD - DEFLECTION CURVES, PIERS 1 AND 3
(1 in. = 2.54 cm, 1 kip = 4.45 kN)

were applied at 60-kip and 100-kip (267-kN and 445-kN) loads for 10 to 15 cycles. Loads above 100 kip (445 kN) were applied in increments of 20 to 40 kip (89 to 178 kN) without unloading. Deflection readings were taken at the completion of each load increment, and the load was left on for a period of about 30 min or until the deflection decreased to less than 0.001 in./min (0.0254 mm/min), whichever occurred earlier.

In all cases, new load increments were applied as fast as it was practical. In general, each load was brought up to the desired value in less than one minute. For cyclic loading, the load was raised to the desired value, deflection and load readings were taken, and the load was removed. In addition to the deflection measurements, tilt measurements were made on Piers 4 and 6 on top of the pier using a surface-mounted tilt transducer.

LOAD TEST RESULTS

Lateral load versus groundline displacement are plotted in Fig. 4 for Test No. 1 (between Piers 1 and 3) and in Fig. 5 for Test No. 2 (between Piers 2 and 3). Typical slope indicator data indicate that the point of rotation of the piers was about 10 ft to 12 ft (3.05 m to 3.66 m) below groundline for loads in excess of 100 kip (445 kN).

For Piers 4 - 7, the measured values of lateral loads versus deflections at the ground surface are plotted in Figs. 6 and 7. The groundline deflections were estimated by adjusting the dial indicator measurements made 6 in. (152 mm) above the groundline on the basis of tilt measurements made at the top of the pier. The groundline deflections are about 90% of the dial indicator readings.

For all piers, the deflection readings taken immediately after each loading and after the deflection had stabilized are shown by two points at the same load. Where unloading was done, all subsequent deflections beyond the last load are referenced to the original zero deflection at the start of the test. The deflection readings after applying the indicated number of cycles are shown with open circles.

COMPARISON OF OBSERVED AND PREDICTED DEFLECTIONS

Since loading for Pier 4 was carried to large deflections, the observed load-deflection data for Pier 4 are compared in Fig. 8 with the predicted values using the procedures suggested by Broms(5), Reese(17), and Poulos (14). The pier data used in the analyses are: pier diameter, $b = 24$ in.; pier length, $L = 18$ ft; and pier

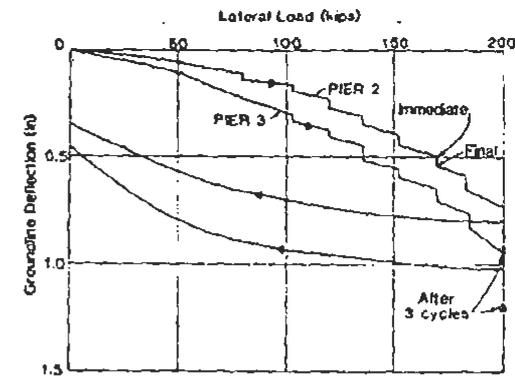


Figure 5. LOAD DEFLECTION CURVES, PIERS 2 AND 3
(1 in. = 2.54 cm, 1 kip = 4.45 kN)

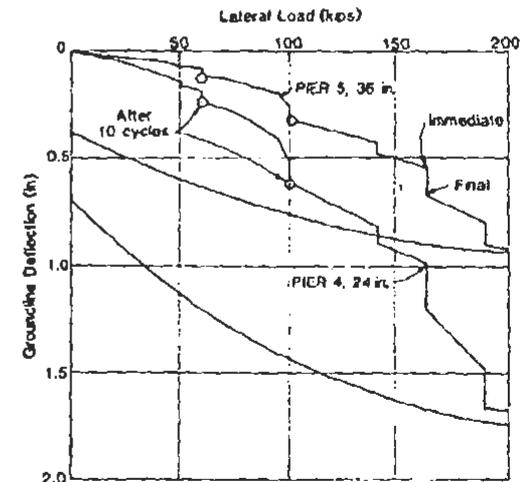


Figure 6. LOAD DEFLECTION CURVES, PIERS 4 AND 5
(1 in. = 2.54 cm, 1 kip = 4.45 kN)

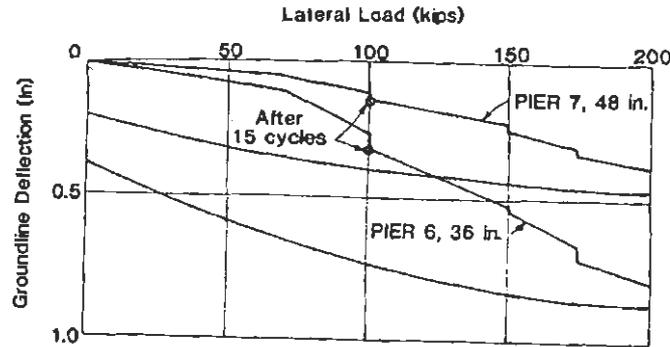


Figure 7. LOAD DEFLECTION CURVES, PIERS 6 AND 7
(1 in. = 2.54 cm, 1 kip = 4.45 kN)

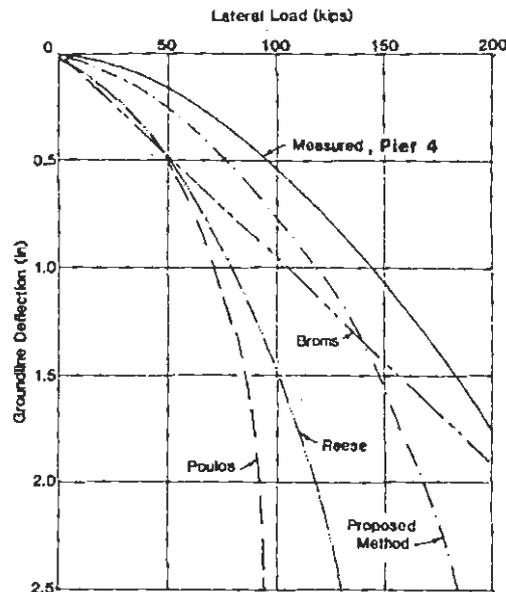


Figure 8. COMPARISON OF COMPUTED
AND MEASURED DEFLECTIONS
(1 in. = 2.54 cm, 1 kip = 4.45 kN)

rigidity, $EI = 7.08 \times 10^{10} \text{ lb-in}^2$ (1 in. = 2.54 cm; 1 ft = 0.305 m; 1 lb-in² = 2.92 kg-cm²).

Broms' method(5) is based on a subgrade-reaction approach and uses non-dimensional deflection coefficients and a constant coefficient of subgrade reaction, n_h , based on the density of sand and the location of the water table only. Broms(5) suggested the use of n_h values proposed by Terzaghi(19). A value of $n_h = 65 \text{ pci}$ (1.8 kg/cm³) for dense sand above the water table was used in the analysis. The linear load-deflection prediction obtained by this procedure is shown in Fig. 8.

Reese (16) used the results of an instrumented load test to propose a semi-empirical procedure for constructing $p-y$ (lateral resistance-pile deflection) curves to be used in a finite difference solution of the differential equation of bending using a computer program (18). The procedure was based on a load test on a 24-in. (610-mm) diameter pipe pile in medium dense sand below the water table. Later Reese (17) extended the recommended procedure to sand above the water table by modifying values of coefficient of subgrade reaction defining the initial part of the $p-y$ curves.

The soil properties used in Reese's $p-y$ analysis(17) were: angle of internal friction, $\phi = 36^\circ$; soil unit weight, $\gamma = 105 \text{ pcf}$; and coefficient $k = n_h = 225 \text{ pci}$. (1 pcf = 16.01 kg/m³; 1 pci = 0.0276 kg/cm³).

Poulos (14) has proposed an elastic continuum approach to the problem using either a constant modulus with depth or a linearly increasing modulus with depth. The non-linear soil response is taken into account by using a yield-deflection factor. Poulos (14) indicates that for a modulus increase may be taken as the coefficient of subgrade reaction n_h in the subgrade reaction analysis and recommends using Terzaghi's (19) values of n_h for computations. A value of N_h (rate of modulus increase with depth) of 65 pci (1.8 kg/cm³) was used in Poulos(14) analysis.

A review of Fig. 8 indicates that the predicted deflections using the existing procedures (5,14,17) in the working load range of 67 kip to 100 kip (298 kN to 445 kN) are generally two to five times the observed deflections. Although Terzaghi(19) made no recommendation concerning the range of loading for which his values of n_h are applicable, it appears that his values are more representative of ultimate conditions (near failure) rather than the working stress range as assumed by Broms(5). Vesic (20) suggests that the coefficient of subgrade reaction

n_h can best be obtained from measured deflections and slopes in a lateral load test. In addition, observations on instrumented piles indicate that n_h is not constant for a given relative density, as assumed by Terzaghi(19), but it varies with lateral deflection of the pier. To take into account the variation of n_h with deflection, analyses were made of the load test data obtained during this testing program and the data available in the literature (1,2,4,8,9,12,16) to develop an approximate relationship for the variation of n_h with the level of deflection. These results form the basis of the proposed method.

THEORETICAL CONSIDERATIONS

Consider a single vertical pier of diameter b , length L , and structural stiffness EI , placed in a soil mass of known characteristics. The statical influences along the pier can be determined by considering the pier as a beam and using the differential equation of bending:

$$EI \frac{d^4 y}{dz^4} = p(z) \dots\dots\dots(1)$$

in which E = modulus of elasticity of pier; I = moment of inertia of pier; y = deflection; $p(z)$ = lateral soil pressure at depth z .

The ratio of lateral soil pressure to the deflection is the subgrade reaction modulus so that:

$$p/y = K_h \dots\dots\dots(2)$$

where K_h = subgrade reaction modulus.

Closed form solutions of Eq. 1 are available (10) for a constant K_h . However, observations on laterally loaded piles in granular soils indicate that a more realistic assumption is a modulus K_h linearly increasing with depth according to:

$$\frac{p}{y} = K_h = n_h z \dots\dots\dots(3)$$

in which n_h = empirical quantity called the coefficient of subgrade reaction.

The solutions of the differential equation, Eq. 1, for K_h linearly increasing with depth have been obtained using the method of finite differences and have been presented as non-dimensional coefficients (11,15). These solutions give deflection as:

$$y = \frac{A_y P T^3}{EI} + \frac{B_y (Pa) T^2}{EI} \dots\dots\dots(4)$$

in which y = deflection; P = lateral load; a = height of application of the applied load P above the ground surface; A_y, B_y = non-dimensional deflection coefficients; and

$$T = \left(\frac{EI}{n_h} \right)^{0.2} \dots\dots\dots(5)$$

T is the characteristic pier length.

For values of L/T greater than 2.0, the values of A_y and B_y for deflection at the ground surface are shown in Fig. 9. These values are based on computations using elastic pile theory (11). Similar coefficients are available for moment, slope, and shear (13,15).

For L/T ratio of less than 2.0, the pier can be assumed to be a short rigid-pier with groundline deflection given by:

$$y = \frac{18 P}{L^2 n_h} + \frac{24 Pa}{L^3 n_h} \dots\dots\dots(6)$$

For $L/T = 2$, the computed deflections using Eq. 4 or Eq. 6 may differ by up to 15% due to the differences in the results of rigid-pile and elastic-pile theory.

PROPOSED METHOD

A review of load test data obtained during this testing program and of data available in the literature (1,2,4,8,9,12,16) indicates that for sands, the values of n_h can be expressed in terms of relative density and normalized deflection (y/b). Based on currently available full-scale load test data, the proposed curves for n_h versus y/b for values of y/b between 0.5% and 10% are shown in Fig. 10 as a function of relative density of sand. The values of n_h shown in Fig. 10 are for sands above the water table. Preliminary comparisons of observed and predicted data indicate that for sands below the water table 50% of the values of n_h given in Fig. 10 may be used. A step-by-step procedure to compute a load-deflection curve for a pier is:

1. Assemble information on pier and loading: diameter, b ; length, L ; modulus of elasticity of pier, E ; moment of inertia of pier, I ; area of steel, A_s ; and height of application of load above ground, a .
2. Assemble available soil data: soil unit weight, γ ; and, relative density, D_r . If significant variations with depth are found, use the average values in the sand to a depth equal to two to three times the pier diameter. The relative density may be estimated using standard penetration blow count and correlations

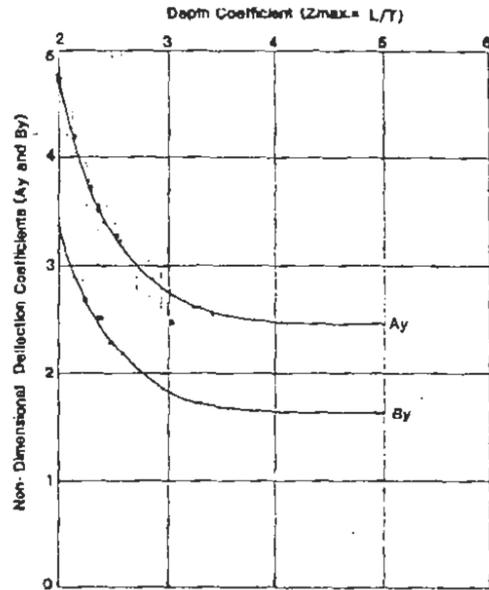


Figure 9. DEFLECTION COEFFICIENTS, A_y AND B_y AT GROUND SURFACE

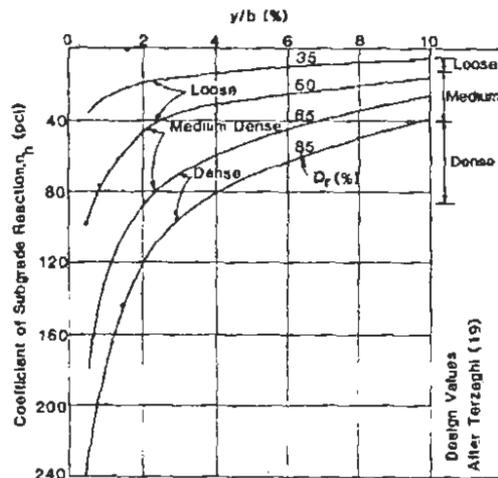


Figure 10. VARIATION OF n_h WITH DENSITY AND DEFLECTION FOR SAND ABOVE WATER TABLE
(1 pci = 0.0276 kg/cm³)

proposed by Bazarra(3) or by using correlations between cone penetration resistance and relative density.

3. Select y/b values, such as 0.5%, 1.0%, 2.5%, 5.0%, 8.0%, and 10.0%.
4. Compute deflection y corresponding to the selected values of y/b .
5. Obtain n_h corresponding to various values of y/b selected in step 3 and relative density selected in step 2, using Fig. 10.
6. Compute characteristic length T using Eq. 5.
7. Compute the ratio $Z_{max} = L/T$.
8. Obtain A_y and B_y from Fig. 9 corresponding to values of L/T in step 7.
9. Compute P using Eq. 4 or 6.

An illustrative example using this procedure for Pier 4 is shown in Appendix III. The computed load-deflection data are shown in Fig. 8 as the proposed method.

REVIEW AND CONCLUSIONS

The proposed method of estimating lateral deflections for drilled piers provides a simple empirical procedure to take into account the dependence of the coefficient of subgrade reaction n_h on the magnitude of deflection. Comparisons of the computed values using the proposed method for the seven tests reported herein with the observed values indicate that the predicted values are generally one to three times the observed values. As a comparison, the existing procedures (5,14,17) generally predict values which are two to more than five times the observed values.

A review of Figs. 4 and 5 indicates that three similar piers constructed within a distance of about 30 ft (9.2 m) had deflections ranging from 0.6 in. to 1.1 in. (15.2 mm to 27.9 mm) at a load of 200 kip (890 kN). These data indicate that variations in soil and construction conditions can result in differences of as much as 100% in observed deflections of similar piers.

Cyclic loads of up to one-half the ultimate load appear to increase the measured deflections for short-term static loading by 20 to 35% for sands above the water table.

The proposed relationship between n_h , relative density, and y/b shown in Fig. 10 is based on a limited number of full-scale load test data. The curves provide a simple procedure to account for non-linear soil response and appear to provide a reasonable prediction of the load-deflection behavior of drilled piers in sand. However, it is recognized that the curves may be modified as more full-scale load test data become available and they should thus be used with caution. No safety factor is included in these curves and an appropriate safety factor must be applied to the loads.

In addition to the allowable deflection, the pier design should consider the ultimate capacity of the pier, especially for short piers with an L/T ratio of 2.0 or less. The structural design should include considerations of stresses in concrete and steel due to the combined effects of axial and lateral loads. For piers with an L/b ratio of less than 4, base shear may be important.

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APPENDIX 1 - REFERENCES

1. Adams, J. I., and Radhakrishna, H. S., "The Lateral Capacity of Deep Augered Footings," Proceedings of the 8th International Conference on Soil Mechanics and Foundation Engineering, Moscow, USSR, 1973, Vol. 2, Part 1, pp. 1-8.
2. Alizadeh, M., and Davisson, M. T., "Lateral Load Tests on Piles - Arkansas River Project," Journal of the Soil Mechanics and Foundation Division, ASCE, Vol. 96, No. SM5, Sept. 1970, pp. 1583-1604.
3. Bazarra, A., "Use of the Standard Penetration Test for Estimating Settlement of Shallow Foundations in Sand," Ph.D. Dissertation, University of Illinois, Civil Engineering, 1967, p. 379.
4. Botea, E., Manolin, I., and Abramescu, T., "Large Diameter Piles Under Axial and Lateral Loads," Proceedings of the 8th International Conference on Soil Mechanics and Foundation Engineering, Moscow, USSR, Vol. 2, Part 1, 1973, pp. 27-32.
5. Broms, B. B., "Lateral Resistance of Piles in Cohesionless Soils," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 90, No. SM3, Proc. Paper 3909, May 1964, pp. 123-156.
6. Chieruzzi, R., "Lateral Pile Load Test, Report No. 169," Southern California Edison Company, Engineering Department, Rosemead, California, Mar., 1963, pp. 1-9.
7. Czerniak, E., "Resistance to Overturning of Single, Short Piles," Journal of the Structural Division, ASCE, Vol. 83, No. ST2, Proc. Paper 118U, Mar., 1957, pp. 1-25.
8. Davisson, M. T., and Salley, J. R., "Lateral Load Test on Drilled Piers," Performance of Deep Foundations, ASTM Spec. Techn. Publ. 444, 1969, pp. 68-63.
9. Gill, H. L., and Domars, K. R., "Displacement of Laterally Loaded Structures in Nonlinearly Responsive Soils," Technical Report R670, Naval Civil Engineering Laboratory, Port Hueneume, Ca., 1970, pp. 1-59.
10. Hetenyi, M., "Beams on Elastic Foundations," Univ. of Michigan Press, Ann Arbor, Mi., 1946, p. 255.
11. Matlock, H., and Reese, L. C., "Generalized Solutions for Laterally Loaded Piles," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 86, No. SM5, Proc. Paper 2626, Oct. 1960, pp. 63-91.
12. Meyer, B. J., and Reese, L. C., "Analysis of Single Piles Under Lateral Loading," Preliminary Review Copy, Research Report 244-1, Center for Highway Research, The University of Texas at Austin, Austin, Tx., Dec., 1979, pp. 1-145.
13. Navdocks DM-7, Bureau of Yards and Docks, Department of the Navy, Design Manual: Soil Mechanics, Foundations, and Earth Structures, Washington, D.C., Feb., 1962, pp. 7-13-13.
14. Poulos, H. G., and Davis, E. H., "Pile Foundation Analysis and Design," Series in Geotechnical Engineering, John Wiley and Sons, New York, 1980, pp. 1-397.
15. Reese, L. C., and Matlock, H., "Non-dimensional Solutions for Laterally Loaded Piles with Soil Modulus Assumed Proportional to Depth," Proc. 8th Texas Conf. Soil Mech. Found. Engrg., 1956, pp. 1-41.

16. Reese, L. C., Cox, W. R., and Koop, F. D., "Analysis of Laterally Loaded Piles in Sand," Paper No. OTC 2080, Proceedings, Sixth Annual Offshore Technology Conference, Houston, Tx., 1974, Vol. 2, pp. 473-483.
17. Reese, L. C., "Laterally Loaded Piles," in Design Construction and Performance of Deep Foundations - A Seminar Series, February to March 1975, Univ. Exten., The College of Engrg., Univ. of Cal., Berkeley, Ca., Aug., 1975, pp. 1-46.
18. Reese, L. C., "Laterally Loaded Piles: Program Documentation," Journal of the Geotechnical Engineering Division, ASCE, Vol. 103, No. GT4, Proc. Paper 12862, Apr., 1977, pp. 287-305.
19. Terzaghi, K., "Evaluation of Coefficients of Subgrade Reaction," Geotechnique, Vol. 5, No. 4, 1955, pp. 297-326.
20. Vesić, A. S., "Design of Pile Foundations", National Cooperative Highway Research Program, Synthesis of Highway Practice 42, Transportation Research Board, National Research Council, Washington, D.C., 1977, pp. 1-68.

APPENDIX II. - NOTATION

a	= eccentricity of load P
A_s	= area of steel
A_y , B_y	= nondimensional deflection coefficients
b	= pier diameter
D_r	= relative density of sand
E	= modulus of elasticity of pier
I	= moment of inertia of pier
k	= constant of subgrade reaction (same as n_h)
k_h	= subgrade reaction modulus
L	= pier length
n_h	= coefficient of subgrade reaction
N	= standard penetration blow count
N_h	= rate of elastic modulus increase with depth
p	= soil pressure
P	= lateral load on pier
$p(z)$	= lateral soil pressure at depth z
T	= characteristic length
z	= depth below ground surface
z_{max}	= depth coefficient
y	= pier deflection
γ	= soil unit weight
ϕ	= angle of internal friction

APPENDIX III. -- EXAMPLE

Problem:

A drilled and cast-in-place pier with a diameter, $b=2$ ft (0.61 m) and length, $L=18$ ft (5.49 m) is constructed in dense sand. The structural rigidity, EI , of the pier = 7.08×10^{10} lb-in² (2.07×10^{11} kg-cm²).

The soil has an average dry density, $\gamma = 105$ pcf (1681 kg/m³), angle of internal friction, $\phi = 36^\circ$, and an average relative density, $D_r = 85\%$ based on blowcount using Bazzarra's (3) correlation. No water table is present.

Compute the load-deflection curve for lateral load applied at ground surface.

Solution: use step-by-step procedure in a tabular form as follows:

Step

1. Pier data:	$b=24$ in.					
	$L=18$ ft					
	$EI=7.08 \times 10^{10}$ lb-in ²					
	$a=0$ in.					
2. Soil data:	$\gamma=105$ pcf					
	$D_r=85\%$					
3. $y/b(\%)$	0.5	1.0	2.5	5.0	8.0	10
4. y, in.	0.12	0.24	0.6	1.2	1.92	2.4
5. n_h , pci	240	180	104	72	48	40
6. T, in	49.4	52.3	58.4	62.8	68.2	70.7
7. L/T	4.37	4.13	3.70	3.44	3.17	3.05
8. A_y	2.42	2.42	2.50	2.55	2.62	2.65
9. P, kip	29	49	85	134	163	181

NOTE:

1 in.	= 2.54 cm
1 ft	= 30.41 cm
1 lb-in. ²	= 2.92 kg-cm ²
1 pci	= 0.0276 kg/cm ³
1 kip	= 4.45 kN
1 pcf	= 16.01 kg/m ³